SELFSIMILAR MOTIONS OF GAS WITH SHOCK WAVES, SPREADING WITH CONSTANT SPEED INTO GAS AT REST

(AFTOMODEL'NYE DVIZHENIIA GAZA S UDARNYMI VOLNAMI, RASPROSTRANIANSHCHIMISIA S POSTOIANNOI SKOROST'IU PO POKOIASHCHEMUSIA GAZA)

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The class of solutions of selfsimilar uniform motions of gas discovered by Sedov [1] in connection with the "theory of planar sections" has led to the study of a number of practically important hypersonic flows, such as the flow around blunt bodies of revolution with profiles in powers of distance from the nose [4], [5], the flow corresponding to a parabolic shock wave in a diffuser [6], etc.

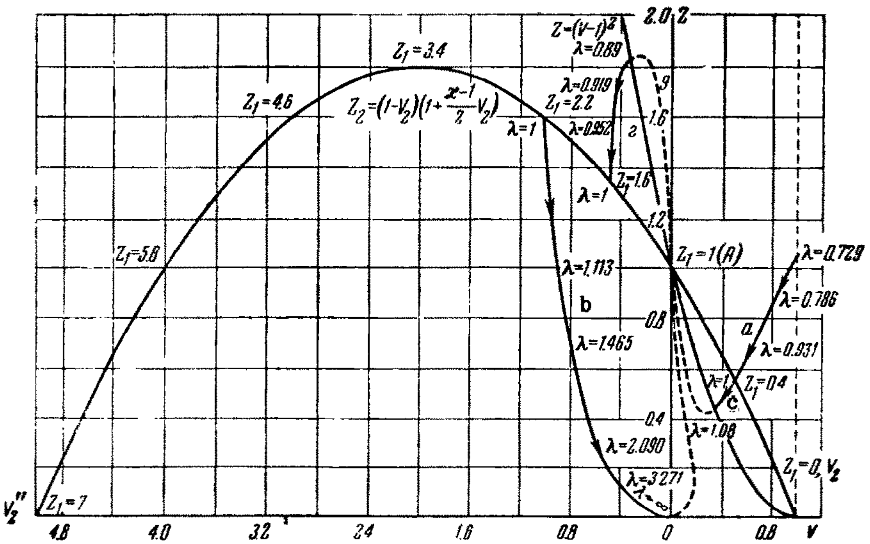

Fig. 1.

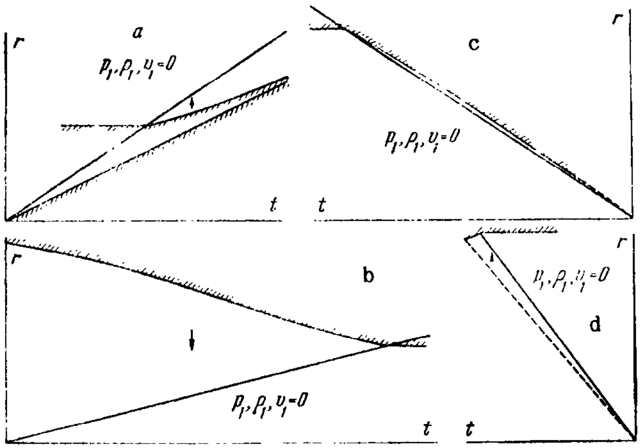

Fig. 2.

In studies of flows in ducts, those with shock waves propagating into gas at rest are of special importance. Sedov [1] has investigated two particular cases of selfsimilar motions when the pressure p_i and the density ρ_i in the quiescent gas are given: (*a*) the problem of a piston which moves with constant speed after a sudden start; (*b*) the problem of focussing on to a point for which the initial velocity of all particles is concentric and of constant magnitude. We will examine all flows in which the shock waves propagate with constant speed u_1. We will seek the variations of the speed v, the density ρ and the pressure p. The independent variables are the distance coordinate r, and the time t, while other dimensionally independent parameters are the density ρ_1 of the gas at rest and the speed u_1 (the dimension of the pressure p_1 being dependent according to $[p_1] = [\rho_1][u_1^2]$). The similarity solution will be a function of the dimensionless combination $\lambda = \beta r / u_1 t$, and is determined from the integration of the system of equations (1) (for the sake of definiteness we discuss the case with cylindrical symmetry):

$$\frac{dZ}{dV} = \frac{Z}{V}\,\frac{(V-1)[(\varkappa+1)V-2]-2Z}{(V-1)^2-2Z} \tag{1}$$

$$\frac{d\ln\lambda}{dV} = \frac{Z-(V-1)^2}{V[(V-1)^2-2Z]}, \qquad R^{\varkappa-1} = cZ\lambda^2$$

where

$$v = \frac{r}{t}V(\lambda), \qquad \rho = \rho_1 R(\lambda),$$

$$p = \frac{\rho_1 r^2}{t^2}P(\lambda), \qquad Z = \frac{\varkappa P}{R}$$

In the quiescent gas ahead of the shock (right-hand corner of Fig. 1 of the V, Z plane) we have:

$$V = 0, \qquad Z_1 = \frac{\varkappa p_1}{\rho_1 u_1^2}$$

while points (V_2, Z_2) corresponding to the possible states immediately behind the shock lie on the segment of the parabola

$$Z_2 = (1 - V_2)\left(1 + \frac{\varkappa - 1}{2} V_2\right) \tag{2}$$

between

$$V'_2 = \frac{2}{\varkappa + 1}, \qquad V_2'' = \frac{2}{1 - \varkappa}$$

The integral curves, which represent the solutions of (1) and therefore the selfsimilar motions of the gas behind the shock wave, all issue from the points on the parabola. Examples of such curves are shown in Fig. 1. Four modes of motion correspond to the class of solutions under investigation: (*a*) flow behind a diverging shock propagating into a quiescent medium [1], (the piston problem, $Z_1 < 1.0$ and $t > 0$); (*b*) flow in front of a diverging shock which leaves the gas quiescent behind itself [1] (the focussing problem, $Z_1 > 1.0$ and $t > 0$); (*c*) flow behind a converging shock propagating into quiescent gas ($Z_1 < 1.0$ and $t < 0$); (*d*) flow into a converging shock behind which the gas comes to rest ($Z_1 > 1.0$ and $t < 0$).

Note. When the intensity of the shock wave, used as parameter, is decreased, the initial points of the integral curves approach point *A* in Fig. 1. The dotted integral curves issuing from *A* correspond to nonstationary isentropic expansion and compression flows. The corresponding stationary axisymmetric flows are found in the supersonic conical flow regions aroung the sterns of ships [7] and in the supersonic conical compression diffusers [8].

Flows of the type (*b*) and (*d*) in the *V*, *Z* plane of Fig. 1 are bounded by the parabola $Z = (V - 1)^2$, which corresponds to the speed of sound. There are corresponding limitations in the physical plane, Fig. 2. For all the cases under consideration, the physical motion is represented in Fig. 2, shock motion by continuous lines and particle (piston) motion by striated lines. Fig. 3 shows the corresponding instantaneous dimensionless spatial distributions of velocity, density, and pressure.

The unsteady cylindrical flows investigated can be utilized to determine steady axisymmetric hypersonic flows on the basis of the "theory of planar sections" [2], [3]. In the unsteady problems the Mach number M_1 characterizing the motion of the shock into the gas at rest is

$$M_1 = \sqrt{\frac{u_1^2 \rho_1}{\varkappa p_1}} = \sqrt{\frac{1}{Z_1}}$$

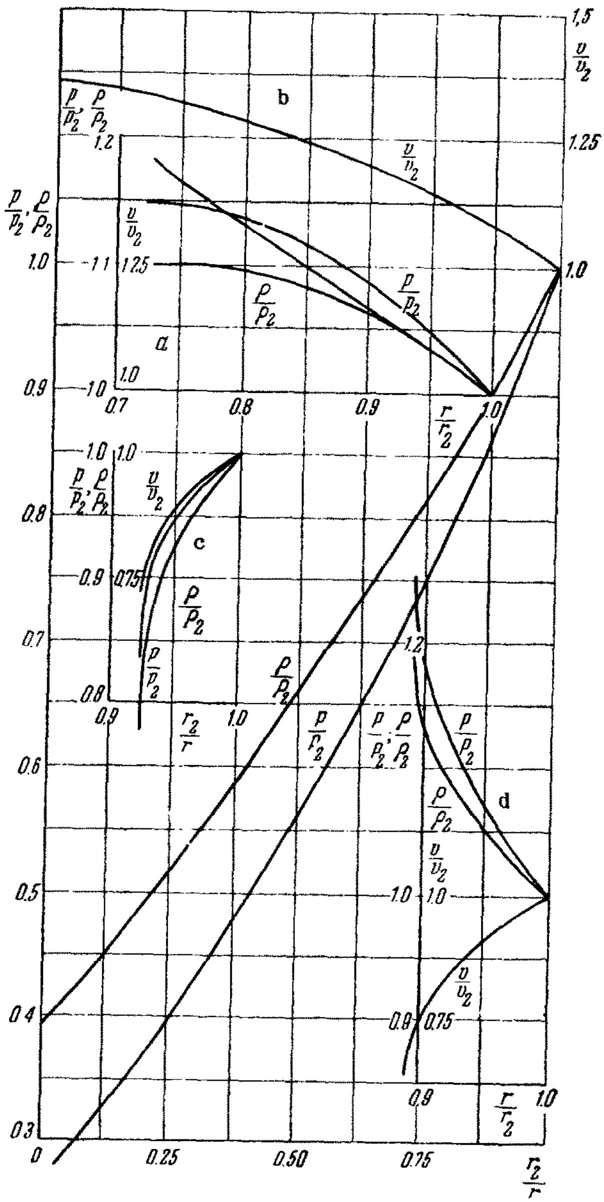

Fig. 3.

In the related stationary problems this same value of M_1 corresponds to the component normal to the shock wave.

The results of computations by this method of cone angle θ_k, shock angle θ_w, and the usual pressure coefficient $p = 2(p_k' - p_1)/\kappa M^2 p_1$ at the cone, for case (*a*) are shown in Fig. 4. As a check on accuracy of the method, comparison is made with exact solutions for the cones (circles in Fig. 4) corresponding to conditions $p_k/p_1 = 3.16$, $r_k/r_2 = .728$, with $Z_1 = .4$ and $M_1 = 1.58$.

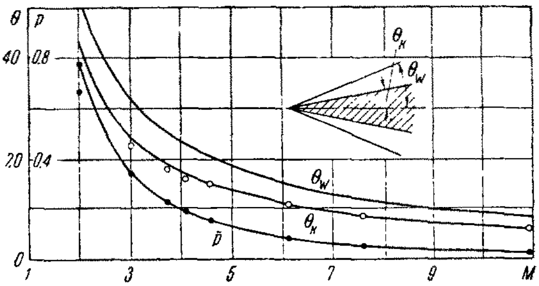

Fig. 4.

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